

Lab 3: Projectile Motion (M3B)

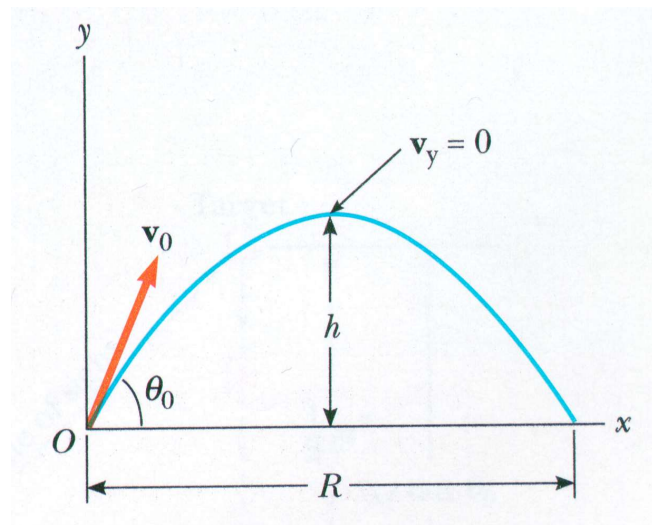
In this lab, you will analyze motion of real objects recorded on a digital video.

Objectives

- Examine motion of a projectile using velocity vs. time graphs.
- Use the equations of kinematics in two dimensions.
- Discover the effect of air resistance on terminal speed.

Projectile Motion (without air resistance)

Projectile motion takes place in two dimensions and can be analyzed as two separate motions. The horizontal component of motion is at constant velocity, whereas the vertical component is at constant acceleration, $a_y = -g = -9.80 \text{ m/s}^2$, just as for a body falling vertically under the action of gravity (a_y is negative because the y-axis is pointing upward).



Equations Describing Projectile Motion (without air resistance)

The position versus time dependence is given by the following formulas for $x(t)$ and $y(t)$ components:

$$x(t) = x_0 + v_0 \cos \theta_0 \cdot t \quad y(t) = y_0 + v_0 \sin \theta_0 \cdot t - \frac{1}{2} g t^2 \quad (1)$$

where (x_0, y_0) is the starting point; θ_0 is the angle of the initial velocity \vec{v}_0 above the horizontal also called the angle of elevation; $v_0 \cos(\theta_0)$ and $v_0 \sin(\theta_0)$ are the horizontal and vertical components of the initial velocity, respectively. If we move the origin of the frame of reference that $x_0 = y_0 = 0$, then

$$x(t) = v_0 \cos \theta_0 \cdot t \Rightarrow t = \frac{x(t)}{v_0 \cos \theta_0} \quad \text{and} \quad y(t) = v_0 \sin \theta_0 \cdot t - \frac{1}{2}gt^2 \quad (2)$$

Solving the equation $x(t)$ for t and substituting into the equation $y(t)$ we obtain the trajectory (i.e., the y versus x graph) of a projectile. It is given by the following equation (observe that this is the equation of a parabola):

$$\begin{aligned} y = y(t) &= v_0 \sin \theta_0 \cdot t - \frac{1}{2}gt^2 = v_0 \sin \theta_0 \cdot \frac{x(t)}{v_0 \cos \theta_0} - \frac{1}{2}g\left(\frac{x(t)}{v_0 \cos \theta_0}\right)^2 = \\ &= \tan \theta_0 \cdot x - \frac{g}{2(v_0 \cos \theta_0)^2} \cdot x^2 \end{aligned} \quad (3)$$

Two components of velocity: $v_x(t)$ and $v_y(t)$ can be found from the slope (as the first derivative or the slope) of the $x(t)$ and $y(t)$ graphs. The horizontal component $v_x(t)$ of the velocity has constant value $v_0 \cos \theta_0$, whereas the vertical component $v_y(t)$ changes at a constant rate typical for the constant acceleration motion. In other words, the expression for $v_y(t)$ shows linear dependence with respect to time.

$$v_x(t) = v_{0x} = v_0 \cos \theta_0 \quad v_y(t) = v_{0y} - gt = v_0 \sin \theta_0 - gt \quad (4)$$

$$|v(t)| = \sqrt{v_x^2(t) + v_y^2(t)} \quad |v_0| = |v(t=0)| = \sqrt{v_{0x}^2 + v_{0y}^2} \quad (5)$$

Projectile motion is just an example of a constant acceleration motion. There are also some other types of the constant acceleration motion, e.g., motion of an object on incline.

The acceleration in the horizontal direction is equal to zero and the vertical component is equal to the acceleration due to gravity. The negative sign of $a_y(t)$ indicates that the acceleration points down, whereas the y -axis is pointing up.

$$a_x(t) = 0 \quad a_y(t) = -g \quad (6)$$

The horizontal range R is equal to the horizontal distance the projectile has traveled when it returns to its initial (launch) height. To find the range R , let us use $R = x(t) - x_0$ and $y(t) - y_0 = 0$ in Eq. (1), obtaining

$$R = (v_0 \cos \theta_0)t \quad \text{and} \quad 0 = (v_0 \sin \theta_0)t - \frac{1}{2}gt^2 \quad (7)$$

Eliminating t between these two equations yields

$$R = \frac{2v_0^2}{g} \sin \theta_0 \cdot \cos \theta_0 = \frac{v_0^2}{g} \sin(2\theta_0) \quad (8)$$

The range R has its maximum value when $\sin(2\theta_0) = 1$, which corresponds to $2\theta_0 = 90^\circ$ or $\theta_0 = 45^\circ$. An application of this could be found in baseball where, for the same amount of force you hit with, the ball travels the furthest when it is struck at an angle of 45° . Launching the

ball, say, at an angle of 25° or at an angle of 65° ($90^\circ - 25^\circ = 65^\circ$) gives the same value of the range, which is smaller than that for 45° . For $\theta_0 = 45^\circ$, the range is equal to $R_{\max} = \frac{v_0^2}{g}$.

To find the maximum height of the trajectory, note that at the highest point the vertical component of the velocity is equal to zero. The projectile is not moving up anymore.

$$v_y(t) = 0 \quad \Rightarrow \quad 0 = v_0 \sin \theta_0 - gt \quad \Rightarrow \quad t = \frac{v_0 \sin \theta_0}{g}$$

The maximum height h can be calculated from the following equation

$$h = (v_0 \sin \theta_0)t - \frac{1}{2}gt^2 = (v_0 \sin \theta_0) \frac{v_0 \sin \theta_0}{g} - \frac{1}{2}g \left(\frac{v_0 \sin \theta_0}{g} \right)^2 = \frac{(v_0 \sin \theta_0)^2}{g} - \frac{(v_0 \sin \theta_0)^2}{2g} \quad (9)$$

$$h = \frac{(v_0 \sin \theta_0)^2}{2g}$$

If the projectile was launched from a different initial height $h_0 = y_0 \neq 0$, instead of the origin, then one needs to add h_0 to the expression above to get the maximum height reached by the body.

$$h = \frac{(v_0 \sin \theta_0)^2}{2g} + h_0 \quad (10)$$

Air resistance

The direction of the air resistance force F_{drag} (also called the air **drag force**) is exactly opposite to the direction of the velocity of the moving object. If an object is falling down, then the drag force is pointing up.

$$F_{drag} = \frac{1}{2}C_D \rho A v^2 \quad \Rightarrow \quad F_{drag} \propto v^2 \quad (11)$$

where C_D ($C_D > 0$) is the drag coefficient, ρ is the air density ($\rho = 1.20 \text{ kg/m}^3$ at 20° C) and A is the cross-sectional area perpendicular to the direction of motion. The drag coefficient depends on the shape of the moving object. For a given object the air drag force is proportional to the speed squared v^2 .

As the object falls, it accelerates due to gravity and the air resistance force F_{drag} increases with velocity. This continues until the air resistance force equals the weight of the falling object. The net force is then equal to zero and the object no longer accelerates, but it falls with a constant velocity. This velocity is called the object's terminal velocity. The absolute value of the terminal velocity is called **terminal speed** v_{term} .

Procedure:

Activity 1: Projectile Motion Without Air Resistance

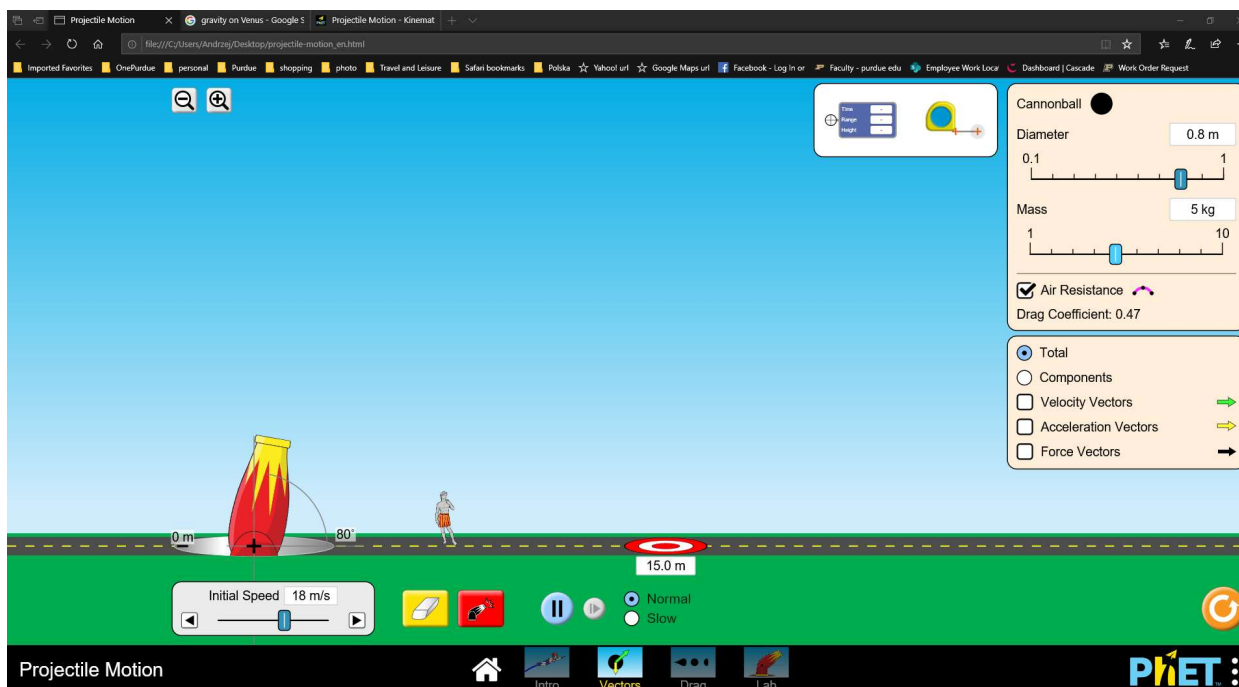
In this experiment, you will use the "Projectile Motion" computer simulation to examine the range of projectiles with and without air resistance.

Setting up the simulation


"PhET Interactive Simulations, University of Colorado Boulder, <https://phet.colorado.edu.>"



The "Projectile Motion" simulation is a part of the collection of interactive simulations called PhET, which was developed at University of Colorado Boulder. To run the "Projectile Motion" simulation, download "*projectile-motion_en*" file from the Brightspace page for Physics 220 lab activities folder. Next, double-click on it to start the simulation. Since this is an html document, it should start the default browser application.

There are multiple modes to run this simulation. We will use the "Lab" mode, so switch to this mode. You should run the simulation in a maximized window (full screen) and here what you should see.




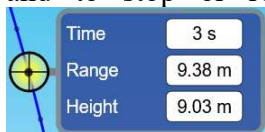
At the bottom of the screen, you should see several slides and buttons that control the

simulation. This  is the slide to adjust the initial speed of the

projectile. Use this button  to launch projectile. The eraser button  would

erase previous runs. The round button  restores the initial, default configuration. This

group of icons  allows users to control the speed of the simulation progress and to stop or restart the simulation playback. We will use the following tool



to measure range, height and time for points on the projectile trajectory.

The other slides adjust the shape, mass and diameter of the projectile. You may turn on or turn off the air resistance and even change the acceleration due to gravity. Another important parameter is the elevation angle. That is the angle between the horizontal and the initial velocity vector. To change the elevation angle, tilt the big gun with the computer mouse.

Set the following initial conditions:

- Initial speed = 15 m/s;
- Initial elevation angle = 25° ;
- Projectile – Pumpkin;
- Mass = 5 kg
- Diameter 0.37 m
- Gravity 9.81 m/s^2
- No air resistance.

Run the simulation, measure the range of the projectile and record the range in the table. Repeat measurements for the same initial speed, but for other values of the elevation angle. Record the results in the table. **The get accurate distance measurements use the “Time/Range/Height” tool and align the crosshair with the last point on the trajectory.**

What is the elevation angle that correspond to the maximum range? What is the value of the maximum range?

Print the graph with multiple trajectories.

Next, set the elevation angle to the value required for the maximum range, do not change the initial speed, but reduce gravity to the value equal to the acceleration due to gravity on planet Venus, i.e., 8.87 m/s^2 . Run the simulation and record the range of the projectile.

Answer the theoretical questions regarding velocity and acceleration in a projectile motion. What is the name of the curve describing the trajectory for the projectile motion without air resistance?

Activity 2: Projectile Motion with Air Resistance

In this activity, you will measure the range in projectile motion with air present.



Erase the previous runs with button.

Change the gravity back to 9.81 m/s^2 and turn on the air resistance for 0 m altitude. All other parameters should be the same as for Activity 1.

Find the elevation angle that gives the maximum range. To get the exact value of the elevation angle, you may need to use varying steps, with the final tune in 1° steps. Record the elevation and the range.

In the second part of this activity, double the diameter of the pumpkin projectile from 0.37 m to 0.74 m. This will increase air resistance effect. Find the elevation angle that gives the maximum range. Record the elevation and the range for that elevation angle.

Print the trajectory for the elevation angle that gives the maximum range with air resistance for the larger pumpkin.

Activity 3: Projectile Motion Range

(1 p.)

Consider the following situation: the initial velocity of the projectile has doubled, but the elevation angle has not changed. How is the range of the projectile going to change? Ignore the air resistance.

To answer this question setup another simulation of projectile motion without air resistance. Measure the range and next double the value of the initial velocity keeping all other parameters unchanged.

What is the observed change of the projectile range?

Make sure to complete the following tasks while in the lab room:

You must submit the answers to the prelaboratory questions online. (3.5 points)

1. Two graphs from Activities 1 and 2. (2 × 1 = 2 points)

2. Your completed Data Sheets. (4.5 points)

3. Return the completed lab report to your lab TA.